Roll No: | | | | | | | | | | | | | |

BTECH
(SEM I) THEORY EXAMINATION 2021-22
ENGINEERING MATHS-I

Time: 3 Hours ***Total Marks: 100***

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. 2 x 10 = 20

a.	Find y_n, if $y = x^2 e^{2x}$.
b.	If $u(x,y) = (x^3 + x^3)^{\frac{1}{5}}$, find the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$.
c.	If $z = u^2 + v^2, u = r\cos\theta, v = r\sin\theta$ find $\frac{\partial z}{\partial r}$.
d.	Prove that $a^x = 1 + x\log a + \frac{x^2}{2!}(\log a)^2 + \frac{x^2}{3!}(\log a)^3 + \cdots \ldots\ldots\ldots\ldots..$
e.	Reduce the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$ into normal form.
f.	Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.
g.	Evaluate $\Gamma\left(-\frac{3}{2}\right)$.
h.	Evaluate $\int_0^{\pi}\int_0^{a\sin\theta} r dr d\theta$.
i.	Show that $\vec{V} = (y+z)\hat{\imath} + (z+x)\hat{\jmath} + (x+y)\hat{k}$ is solenoidal.
j.	Prove that $div(\emptyset\vec{a}) = \emptyset\, div\vec{a} + (grad\emptyset).\vec{a}$

SECTION B

2. Attempt any *three* of the following: 10x3=30

a.	If $y = (sin^{-1}x)^2$, show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$ and calculate $y_n(0)$.
b.	A rectangular box, open at the top, is to have given capacity. Find the dimensions of the box requiring least material for its construction.
c.	Reduce the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ *to the diagonal form.*
d.	Show that $\iiint \frac{dxdydz}{(x+y+z)^3} = \frac{1}{2}\log 2 - \frac{5}{16}$. The integral being taken throughout the volume bounded by $x = 0, y = 0, z = 0, x + y + z = 0$.
e.	Verify Stoke's theorem for $\vec{F} = x^2\hat{\imath} + xy\hat{\jmath}$ integrated round the square whose sides are $x = 0, y = 0, x = a, y$ in the plane $z = 0$.

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SECTION C

3. Attempt any *one* part of the following: 10x1=10

a.	Trace the curve: $x^3 + y^3 - 3axy = 0$.
b.	If $u = \frac{x^3y^3z^3}{x^3+y^3+z^3} + \log\left(\frac{xy+yz+zx}{x^2+y^2+z^2}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 6\frac{x^3y^3z^3}{x^3+y^3+z^3}$.

4. Attempt any *one* part of the following: 10x1=10

a.	Expand $x^2y + 3y - 2$ in powers of $(x-1)$ *and* $(y+2)$ using Taylor's Theorem.
b.	If u, v and w are the roots of $(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$, cubic in λ, find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

5. Attempt any *one* part of the following: 10x1=10

a.	Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9, 7x + 3y - 2z = 8, 2x + 3y + \lambda z = \mu$ have (a) no solution, (b) a unique solution and (c) an infinite number of solutions.
b.	Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ and hence find A^{-2}.

6. Attempt any *one* part of the following: 10x1=10

a.	Find the arc length of the curve $y = \sqrt{x}$ from $x = 0$ *to* $x = 4$.
b.	Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate the same.

7. Attempt any *one* part of the following: 10x1=10

a.	Find the directional derivative of $\emptyset = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$ at the point $P(3,1,2)$ in the direction of the vector $yz\hat{i} + zx\hat{j} + xy\hat{k}$.
b.	Verify Green's theorem in the plane for $\oint_C [(x^2 - 2xy)dx + (x^2y + 3)dy]$ where C is the boundary of the region defined by $y^2 = 8x$ and $x = 2$.