Subject Code: KAS103T

PAPER ID-311288

Roll No:

B.TECH
(SEM I) THEORY EXAMINATION 2020-21
ENGINEERING MATHEMATICS-I

Time: 3 Hours *Total Marks: 100*

Note: 1. Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt *all* questions in brief. **2 x 10 = 20**

Qno.	Question	Marks	CO
a.	Prove that the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary.	2	1
b.	State Rank-Nullity Theorem.	2	1
c.	State Rolle's Theorem.	2	2
d.	Discuss all the symmetry of the curve $x^2y^2 = x^2 - a^2$	2	2
e.	If $u = f(y-z, z-x, x-y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$	2	3
f.	If $x = e^v \sec u, y = e^v \tan u$, then evaluate $\frac{\partial(x,y)}{\partial(u,v)}$.	2	3
g.	Evaluate $\int_0^1 \int_0^{x^2} e^{y/x}\, dydx$.	2	4
h.	Calculate the volume of the solid bounded by the surface x = 0, y =0, x+y+z=1 and z=0.	2	4
i.	Show that the vector $\vec{V} = (x+3y)\hat{\imath} + (y-3z)\hat{\jmath} + (x-2z)\hat{k}$ is solenoidal.	2	5
j.	State Green's theorem.	2	5

SECTION B

2. Attempt any *three* of the following:

Qno.	Question	Marks	CO
a.	Find the inverse of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$	10	1
b.	If $y = e^{\tan^{-1}x}$, prove that. $(1 + x^2)y_{n+2} + [(2n+2) x-1) y_{n+1} + n (n+1) y_n = 0$.	10	2
c.	If $u^3+v+w = x+y^2+z^2$, $u+v^3+w = x^2+y+z^2$, $u+v+w^3 = x^2+y^2+z$,Show that: $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1-4xy(xy+yz+zx)+16xyz}{2-3(u^2+v^2+w^2)+27u^2v^2w^2}$	10	3
d.	Evaluate by changing the variables, $\iint_R (x+y)^2\, dx\, dy$ where R is the region bounded by the parallelogram x+y=0, x+y =2, 3x-2y=0 and 3x-2y = 3.	10	4
e.	Use divergence theorem to evaluate the surface integral $\iint_S (xdydz + ydzdx + zdxdy)$ where S is the portion of the plane x+2y+3z=6 which lies in the first octant.	10	5

SECTION C

3. Attempt any *one* part of the following:

Qno.	Question	Marks	CO
a.	Find non-singular matrices P and Q such that PAQ is normal form. $\begin{bmatrix}1 & 1 & 2\\1 & 2 & 3\\0 & 1 & 1\end{bmatrix}$	10	1
b.	Find the eigen values and the corresponding eigen vectors of the following matrix. $A = \begin{bmatrix}2 & 0 & 1\\0 & 3 & 0\\1 & 0 & 2\end{bmatrix}$.	10	1

4. Attempt any *one* part of the following:

Qno.	Question	Marks	CO
a.	Find the envelope of the family of lines $\frac{x}{a}+\frac{y}{b}=1$, where $a\ and\ b$ are connected by the relation $a^n+b^n=c^n$	10	2
b.	If y = sin (m $\sin^{-1}$x), find the value of y_n at x= 0.	10	2

5. Attempt any *one* part of the following:

Qno.	Question	Marks	CO
a.	Divide 24 into three parts such that continued product of first,square of second and cube of third is a maximum.	10	3
b.	If $u = sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$,prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2\cot u$. Also evaluate $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$.	10	3

6. Attempt any *one* part of the following:

Qno.	Question	Marks	CO
a.	Evaluate the following integral by changing the order of integration $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}dydx$.	10	4
b.	A triangular thin plate with vertices (0,0),(2,0) and (2,4) has density $\rho = 1+x+y$. Then find: (i) The mass of the plate. (ii) The position of its centre of gravity G.	10	4

7. Attempt any *one* part of the following:

Qno.	Question	Marks	CO
a.	A fluid motion is given by $\vec{v} = (y\sin z - \sin x)\hat{\imath} + (x\sin z + 2yz)\hat{\jmath} + (xy\cos z + y^2)\hat{k}$.Is the motion irrotational? If so, find the velocity potential.	10	5
b.	Verify Stoke's theorem for the function $\vec{F} = x^2\hat{\imath} + xy\hat{\jmath}$ integrated round the square whose sides are x=0,y=0,x=a,y=a in the plane z=0.	10	5