

B. TECH.
(SEM IV) THEORY EXAMINATION 2022-23
ENGINEERING MATHEMATICS IV

Time: 3 Hours

Total Marks: 100

Note: Attempt all Sections. If require any missing data; then choose suitably.

SECTION A

1. Attempt all questions in brief.

2 x 10 = 20

- (a) Obtain a partial differential equation that governs the family of surfaces

$$z = (x - \alpha)^2 + (y - \beta)^2.$$

- (b) Find the complete integral of the partial differential equation

$$z = px + qy + \frac{p}{p+q}, p = \frac{\partial z}{\partial x} \text{ \& } q = \frac{\partial z}{\partial y}.$$

- (c) Classify the partial differential equation

$$r + 2s + (\sin^2 x) t + q = 0, r = \frac{\partial^2 z}{\partial x^2}, s = \frac{\partial^2 z}{\partial y^2} \text{ \& } t = \frac{\partial^2 z}{\partial x \partial y}$$

- (d) Write down the two dimensional heat equation.

- (e) What is the relation between the regression coefficients and the coefficient of correlation?

- (f) The fourth central moment is 48. What must be its standard deviation in order that the distribution be mesokurtic.

- (g) A and B are any two independent events such that $P(A) = 0.4$, $P(A \cup B^c) = 0.7$. Find the $P(B)$, where B^c is the complementary event of event B.

- (h) The random variable X is said to follow the Normal distribution with mean 9 and standard deviation 3, find x^* such that $P(X > x^*) = 0.16$.

- (i) Write down the definition of the null hypothesis.

- (j) What is Statistical Quality Control (SQC)? Define in brief.

SECTION B

2. Attempt any three of the following:

10x3=30

- (a) Find the general solution of the partial differential equation

$$(y + z)p + (z + x)q = (x + y).$$

- (b) Solve the partial differential equation by the method of separation of variables $4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$,

$$\text{given that } u = 5e^{-y} - e^{-5y}, \text{ when } x = 0.$$

- (c) Use the method of least squares to fit the curve $y = ab^x$ for the following data

x	2	3	4	5	6
y	144	172.8	207.4	248.8	298.5

- (d) In a normal distribution, 12% of the items are under 30 and 85% items are under 60. Find the mean and standard deviation.
- (e) The annual rainfall in Lucknow city is normally distributed with mean 45 cm. The rainfall during the last five years are 48 cm, 42 cm, 40 cm, 44 cm and 43 cm respectively. Can we conclude that the average rainfall during the last five years is less than the normal rainfall? Test at 5% level of significance. [The tabulated value of $t_{0.05} = 2.776$ and $t_{0.1} = 2.132$ for 4 degree of freedom.]

SECTION C

3. Attempt any one part of the following: 10x1=10

- (a) Find the solution of the partial differential equation

$$\left[2D^2 + 5DD' + 3(D')^2\right]z = ye^x, D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}.$$

- (b) Find the complete integral of the partial differential equation

$$(p^2 + q^2)x = pz.$$

4. Attempt any one part of the following: 10x1=10

- (a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = a \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest from this position, find the displacement.
- (b) An insulated rod of length l has its ends A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced at 0°C and maintained at 0°C , find the temperature at a distance x from A at time t .

5. Attempt any one part of the following: 10x1=10

- (a) If $4x - 5y + 33 = 0$ and $20x - 9y = 107$ are two lines of regression. Find the mean values of x and y , the coefficient of correlation and the standard deviation of y if the variance of x is 9.
- (b) First four moments about 2 are 1, 2.5, 5.5 and 16 respectively. Find the first four central moments, moments about origin and coefficient of skewness.

6. Attempt any one part of the following: 10x1=10

- (a) A bag A contains 8 white and 4 black balls. A second bag B contains 5 white and 6 black balls. One ball is drawn at random from bag A and is placed in bag B. Now, a ball is drawn at random from bag B. It is found that this ball is white. Find the probability that a black ball has been

transferred from bag A.

- (b) If X variable follow the Poisson distribution such that $P(X = 2) = 9P(X = 4) + 90P(X = 6)$. Find mean, variance and distribution.

7. Attempt any one part of the following:

10x1=10

- (a) In an experiment on pea breeding the following frequency of seeds were obtained:

Red & Yellow	Wrinkled & Yellow	Round & Green	Wrinkled & Green	Total
315	101	108	32	556

Theory predicts the frequencies should be in the proportions 9:3:3:1. Examine the correspondence between theory and experiment. Test at 5% level of significance. [The tabulated value of $\chi^2_{0.05} = 7.815$ for 3 degree of freedom.]

- (b) The given table shows that the value of sample mean $\bar{X}$ and the range R for 10 samples of size 5 each. Draw mean and range chart and also comment on the state of control of the process.

(Given $A_2 = 0.58, D_3 = 0, D_4 = 2.115$).

Sample No.	1	2	3	4	5	6	7	8	9	10
$\bar{X}$	45	46	48	52	53	37	51	46	47	38
R	4	5	6	7	4	5	7	6	6	4